12-4 Operations with Events

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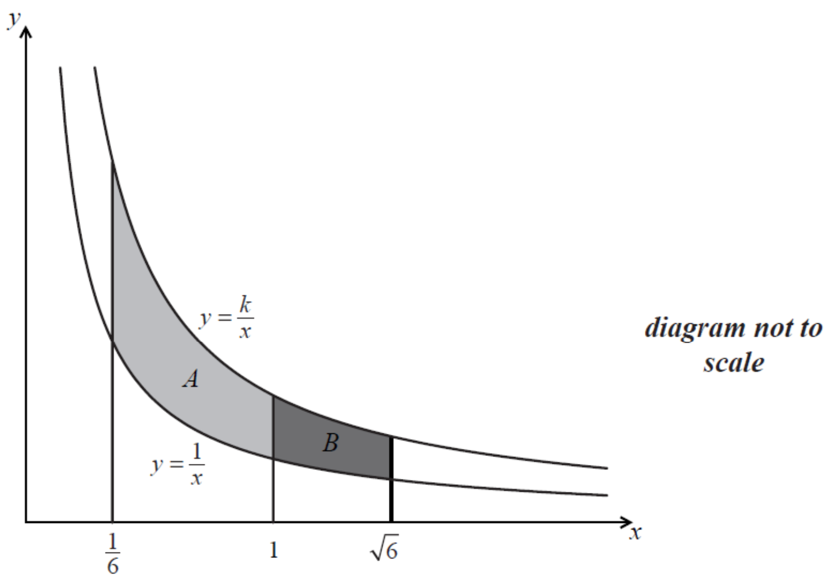
6. *[Maximum mark: 8]* M12/5/MATHL/HP1/ENG/TZ1/XX

The graph below shows the two curves $y = \frac{1}{x}$ and $y = \frac{k}{x}$, where $k > 1$.



(a) Find the area of region A in terms of k. *[3 marks]*

(b) Find the area of region B in terms of k. *[2 marks]*

(c) Find the ratio of the area of region A to the area of region B. *[3 marks]*

6. (a) $\int_{\frac{1}{6}}^{1} \frac{k}{x} - \frac{1}{x}\,\mathrm{d}x = (k-1)[\ln x]_{\frac{1}{6}}^{1}$ ***M1A1***

Note: Award ***M1*** for $\int \frac{k}{x} - \frac{1}{x}\,\mathrm{d}x$ or $\int \frac{1}{x} - \frac{k}{x}\,\mathrm{d}x$ and ***A1*** for $(k-1)\ln x$ seen in part (a) or later in part (b).

$= (1-k)\ln\frac{1}{6}$ ***A1***

[3 marks]

(b) $\int_{1}^{\sqrt{6}} \frac{k}{x} - \frac{1}{x}\,\mathrm{d}x = (k-1)[\ln x]_{1}^{\sqrt{6}}$ ***(A1)***

Note: Award ***A1*** for correct change of limits.

$= (k-1)\ln\sqrt{6}$ ***A1***

[2 marks]

(c) $(1-k)\ln\frac{1}{6} = (k-1)\ln 6$ ***A1***

$(k-1)\ln\sqrt{6} = \frac{1}{2}(k-1)\ln 6$ ***A1***

Note: This simplification could have occurred earlier, and marks should still be awarded.

ratio is 2 (or 2:1) ***A1***

[3 marks]

Total [8 marks]

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Probability Rules

- Rule # 1 – The probability of an event occurring is always between 0 and 1.

$$0 \le P(A) \le 1$$

- Rule # 2 – The sum of the probabilities of all possible outcomes must = 1. P(S) = 1

- Rule #3 – If two events have no outcomes in common, the probability that one or the other occurs is the sum of their individual probabilities.

$$P(A \cup B) = P(A) + P(B)$$

- Rule #4 – The sum of the probabilities of event A and event A' is 1.

$$P(A') = 1 - P(A)$$

Probability Rules

Rule # 5 – For any 2 events A & B

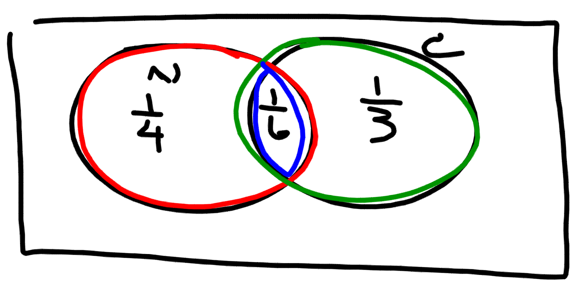
$$P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B)$$

Ex1. A chocolate is selected randomly from a box. The probability of it containing a nut is 1/4. The probability of it containing a caramel is 1/3. The probability of it containing both a nut and a caramel is 1/6. What is the probability of selecting a chocolate containing a nut, or caramel, or both?

$$P(N \cup C) = P(N) + P(C) - P(N \cap C)$$

$$= \frac{1}{4} + \frac{1}{3} - \frac{1}{6}$$

$$= \frac{3}{12} + \frac{4}{12} - \frac{2}{12} = \frac{5}{12}$$



Ex2. 13/20 of the students in a school play sports or music. The probability of a randomly selected student playing a sport is 3/5 and the probability of student being in music is 5/8. What percentage of students are in both music and sports?

$P(M) + P(S) - P(M \cap S) = P(M \cup S)$

$3/5 + 5/8 - P(M \cap S) = 13/20$

$24/40 + \frac{25}{40} - P(M \cap S) = 26/40$

$23/40$

Ex3. In a survey 60% of people are in favor of building a new high school and 85% are in favor of building a new stadium. Half of those surveyed would like both a high school and a new stadium. What percentage supported neither a new high school nor a new stadium?

$P(Sc \cup St) = P(Sc) + P(St) - P(Sc \cap St)$

$60/100 + 85/100 - 50/100$

$95/100 = 145/100 - 50/100$

$1 - 95/100 = 5/100 = 5\%$

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Two events A and B are independent if knowing that one of them occurs does NOT change that probability that the other occurs.

Flip a coin and roll a die. These are independent events – getting a tail on the coin does not change the probability of rolling a 5 on the die.

A baby is born. You record the hair color and the eye color. These are NOT independent events.

– Knowing the hair color DOES change the probability of the eye color.

Probability Rules

- Rule # 6 – Multiplication Rule for Independent Events – If 2 events A and B are independent, then

$$P(A \cap B) = P(A) \cdot P(B)$$

$P(5) = \frac{1}{6}$

$P(T) = \frac{1}{2}$

$P(5 \cap T) = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$

D

C

	1	2	3	4	5	6
H	H, 1	H, 2	H, 3			
T		T, 2		T, 4		

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A calc class has 15 girls and 10 boys. 5 boys are juniors and 5 are seniors. 10 girls are juniors and 5 are seniors. A student is selected at random

P(Junior)= $\frac{15}{25} = \frac{3}{5}$

But if you first select a girl, then the probability of choosing a junior changes to:

P(Junior/Girl) = $\frac{10}{15} = \frac{2}{3}$

These are not independent events.

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